OCR
RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4757 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Option 1: Vectors

1 A mine contains several underground tunnels beneath a hillside. The hillside is a plane, all the tunnels are straight and the width of the tunnels may be neglected. A coordinate system is chosen with the z-axis pointing vertically upwards and the units are metres. Three points on the hillside have coordinates A(15, –60, 20), B(–75, 100, 40) and C(18, 138, 35.6).

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence show that the equation of the hillside is $2x - 2y + 25z = 650$. **[5]**

The tunnel T_A begins at A and goes in the direction of the vector $15\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$; the tunnel T_C begins at C and goes in the direction of the vector $8\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$. Both these tunnels extend a long way into the ground.

(ii) Find the least possible length of a tunnel which connects B to a point in T_A. **[6]**

(iii) Find the least possible length of a tunnel which connects a point in T_A to a point in T_C. **[6]**

(iv) A tunnel starts at B, passes through the point $(18, 138, p)$ vertically below C, and intersects T_A at the point Q. Find the value of p and the coordinates of Q. **[7]**

Option 2: Multi-variable calculus

2 You are given that $g(x, y, z) = x^2 + 2y^2 - z^2 + 2xz + 2yz + 4z - 3$.

(i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. **[3]**

The surface S has equation $g(x, y, z) = 0$, and P(–2, –1, 1) is a point on S.

(ii) Find an equation for the normal line to the surface S at the point P. **[3]**

(iii) A point Q lies on this normal line and is close to P. At Q, $g(x, y, z) = h$, where h is small. Find the constant c such that $PQ \approx c|h|$. **[5]**

(iv) Show that there is no point on S at which the normal line is parallel to the z-axis. **[5]**

(v) Given that $x + y + z = k$ is a tangent plane to the surface S, find the two possible values of k. **[8]**

Option 3: Differential geometry

3 A curve has parametric equations

$$x = a(1 - \cos^3\theta), \quad y = a\sin^3\theta, \quad \text{for } 0 \leqslant \theta \leqslant \frac{\pi}{3},$$

where a is a positive constant.

The arc length from the origin to a general point on the curve is denoted by s, and ψ is the acute angle defined by $\tan\psi = \frac{\mathrm{d}y}{\mathrm{d}x}$.

(i) Express s and ψ in terms of θ, and hence show that the intrinsic equation of the curve is

$$s = \frac{3}{2}a\sin^2\psi. \qquad \textbf{[9]}$$

(ii) For the point on the curve given by $\theta = \frac{\pi}{6}$, find the radius of curvature and the coordinates of the centre of curvature. **[9]**

(iii) Find the area of the curved surface generated when the curve is rotated through 2π radians about the y-axis. **[6]**

Option 4: Groups

4 **(i)** Show that the set $P = \{1, 5, 7, 11\}$, under the binary operation of multiplication modulo 12, is a group. You may assume associativity. **[4]**

A group Q has identity element e. The result of applying the binary operation of Q to elements x and y is written xy, and the inverse of x is written x^{-1}.

(ii) Verify that the inverse of xy is $y^{-1}x^{-1}$. **[2]**

Three elements a, b and c of Q all have order 2, and $ab = c$.

(iii) By considering the inverse of c, or otherwise, show that $ba = c$. **[2]**

(iv) Show that $bc = a$ and $ac = b$. Find cb and ca. **[4]**

(v) Complete the composition table for $R = \{e, a, b, c\}$. Hence show that R is a subgroup of Q and that R is isomorphic to P. **[4]**

The group T of symmetries of a square contains four reflections A, B, C, D, the identity transformation E and three rotations F, G, H. The binary operation is composition of transformations. The composition table for T is given below.

	A	B	C	D	E	F	G	H
A	E	G	H	F	A	D	B	C
B	G	E	F	H	B	C	A	D
C	F	H	E	G	C	A	D	B
D	H	F	G	E	D	B	C	A
E	A	B	C	D	E	F	G	H
F	C	D	B	A	F	G	H	E
G	B	A	D	C	G	H	E	F
H	D	C	A	B	H	E	F	G

(vi) Find the order of each element of T. **[3]**

(vii) List all the proper subgroups of T. **[5]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

A 'random walk' is modelled as a Markov chain with five states A, B, C, D, E representing the possible positions, from left to right, of an object. At each 'step' the object moves as follows.

- If the object is at A, it moves one place to the right (to B).
- If the object is at E, it moves one place to the left (to D).
- Otherwise, the probability that the object moves one place to the left is 0.4, and the probability that it moves one place to the right is 0.6.

Steps occur at intervals of one minute, and the time taken to move may be neglected. The object starts at A, so after the first step (one minute later) the object is at B.

(i) Which of the five states are reflecting barriers? **[1]**

(ii) Write down the transition matrix $\mathbf{P}$. **[2]**

(iii) State the possible positions of the object after 10 steps, and give the probabilities that the object is in each of these positions. **[4]**

(iv) Find the probability that after 15 steps the object is in the same position as it was after 13 steps. **[3]**

(v) Find the number of steps after which the probability that the object is at D exceeds 0.69 for the first time. **[3]**

(vi) Find the limits of $\mathbf{P}^{2n}$ and $\mathbf{P}^{2n+1}$ as the positive integer n tends to infinity. **[4]**

(vii) For the interval of 100 minutes between the 200th step and the 300th step, find the expected length of time for which the object is at each of the five positions. **[3]**

(viii) At a certain instant, the object arrives at D. Find the expected number of successive occasions that the object moves to E (and then back to D). Hence find the expected time after this instant when the object first moves to C. **[4]**

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

1 (i)	

1 (ii)	

1 (iii)	

1 (iv)	

2 (i)	
2 (ii)	

2 (iii)	

2 (iv)	

2 (v)	

3 (i)

3 (ii)	

3 (iii)	

4 (i)	
4 (ii)	
4 (iii)	

4 (iv)

4 (v)

R	e	a	b	c
e				
a				
b				
c				

4 (vi)	
4 (vii)	

5 (i)	
5 (ii)	
5 (iii)	

5 (iv)	
5 (v)	

5 (vi)	

5 (vii)	
5 (viii)	
	(answer space continued overleaf)

5 (viii)	(continued)

GCE

Mathematics (MEI)

Advanced GCE

Unit **4757:** Further Applications of Advanced Mathematics

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$\begin{pmatrix}-90\\160\\20\end{pmatrix}\times\begin{pmatrix}3\\198\\15.6\end{pmatrix}=\begin{pmatrix}-1464\\1464\\-18300\end{pmatrix}$ $\left[=-732\begin{pmatrix}2\\-2\\25\end{pmatrix}\right]$	M1 A2	Evaluation of vector product Give A1 for one component correct	A1 for any non-zero multiple correctly obtained
			$2x-2y+25z=2(15)-2(-60)+25(20)$	M1	For $2x-2y+25z=d$	
			Equation of ABC is $2x-2y+25z=650$	E1 **[5]**	Evidence of substitution required	
1	(ii)		$\overrightarrow{AB}\times\mathbf{d}_A=\begin{pmatrix}-90\\160\\20\end{pmatrix}\times\begin{pmatrix}15\\14\\-2\end{pmatrix}=\begin{pmatrix}-600\\120\\-3660\end{pmatrix}$	M1 A2	Appropriate vector product Give A1 if one error	
			$\left\lvert\overrightarrow{AB}\times\mathbf{d}_A\right\rvert=\sqrt{600^2+120^2+3660^2}$ $\lvert\mathbf{d}_A\rvert=\sqrt{15^2+14^2+2^2}$	M1		
			Distance is $\dfrac{\left\lvert\overrightarrow{AB}\times\mathbf{d}_A\right\rvert}{\lvert\mathbf{d}_A\rvert}$	M1		
			Distance is 180 m	A1 **[6]**		
		OR	$\left[\begin{pmatrix}15+15\lambda\\-60+14\lambda\\20-2\lambda\end{pmatrix}-\begin{pmatrix}-75\\100\\40\end{pmatrix}\right]\bullet\begin{pmatrix}15\\14\\-2\end{pmatrix}=0$		M1A1	
			$\lambda=2$		M1 Obtaining a value of λ A1	
			Distance is $\sqrt{(120)^2+(-132)^2+(-24)^2}$		M1	
			Distance is 180 m		A1	

Question			Answer	Marks	Guidance	
1	(iii)		$\mathbf{d}=\begin{pmatrix}15\\14\\-2\end{pmatrix}\times\begin{pmatrix}8\\7\\-2\end{pmatrix}=\begin{pmatrix}-14\\14\\-7\end{pmatrix}\quad [=7\begin{pmatrix}-2\\2\\-1\end{pmatrix}]$	M1 A2	Vector product of direction vectors Give A1 if one error	
			$\overrightarrow{AC}\,.\,\hat{\mathbf{d}}=\dfrac{\begin{pmatrix}3\\198\\15.6\end{pmatrix}\bullet\begin{pmatrix}-2\\2\\-1\end{pmatrix}}{\sqrt{2^2+2^2+1^2}}=\dfrac{374.4}{3}$	M1 M1	Appropriate scalar product Fully correct method for finding distance	
			Distance is 124.8 m	A1 **[6]**		
1	(iv)		$\begin{pmatrix}15\\-60\\20\end{pmatrix}+\lambda\begin{pmatrix}15\\14\\-2\end{pmatrix}=\begin{pmatrix}-75\\100\\40\end{pmatrix}+\mu\begin{pmatrix}93\\38\\p-40\end{pmatrix}$	M1	Must use different parameters	
			$15+15\lambda=-75+93\mu$ $-60+14\lambda=100+38\mu$	A1	Both equations correct	
			$\lambda=25,\quad \mu=5$	M1	Obtaining value of λ or μ	
			$20-50=40+5(p-40)$	M1	*Or other method for finding p*	
			$p=26$	A1		
			Q is $(15+375,\ -60+350,\ 20-50)$	M1		
			Q is $(390,\ 290,\ -30)$	A1 **[7]**		
2	(i)		$\dfrac{\partial g}{\partial x}=2x+2z$	B1		
			$\dfrac{\partial g}{\partial y}=4y+2z$	B1		
			$\dfrac{\partial g}{\partial z}=-2z+2x+2y+4$	B1 **[3]**		

Question			Answer	Marks	Guidance	
2	(ii)		At P, $\frac{\partial g}{\partial x}=-2,\ \frac{\partial g}{\partial y}=-2,\ \frac{\partial g}{\partial z}=-4$	B1		
			Normal line is $\mathbf{r}=\begin{pmatrix}-2\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$	M1 A1 **[3]**	For direction of normal line FT	*Condone omission of* $\mathbf{r}=$
2	(iii)		If Q is $(-2+\lambda,\ -1+\lambda,\ 1+2\lambda)$			
			$h=\delta g\approx\frac{\partial g}{\partial x}\delta x+\frac{\partial g}{\partial y}\delta y+\frac{\partial g}{\partial z}\delta z$	M1	*Requires some substitution in RHS or h on LHS*	Allow M1 for $\delta x=-2+\lambda$ etc
			$=(-2)(\lambda)+(-2)(\lambda)+(-4)(2\lambda)\quad[=-12\lambda]$	A1	FT	
			$\text{PQ}=\sqrt{(\lambda)^2+(\lambda)^2+(2\lambda)^2}$	M1		
			$=\sqrt{6}\,\lvert\lambda\rvert$	A1	FT Allow $\sqrt{6}\,\lambda$	
			$c=\frac{\sqrt{6}}{12}$	A1 **[5]**	A0 for $c=-\frac{\sqrt{6}}{12}$	
2	(iv)		Require $\frac{\partial g}{\partial x}=\frac{\partial g}{\partial y}=0$	M1		
			$x=-z,\quad y=-\frac{1}{2}z$			
			$z^2+\frac{1}{2}z^2-z^2-2z^2-z^2+4z-3=0$	M1	Obtaining equation in one variable	
			$5z^2-8z+6=0$	A1	Or $5x^2+8x+6=0$ or $10y^2+8y+3=0$ or $\lambda^2+14=0$	($\lambda=\frac{\partial g}{\partial z}$)
			Discriminant is $64-120=-56<0$	M1	*Dependent on quadratic with negative discriminant*	
			Hence there are no such points	E1 **[5]**	Correctly shown	

Question			Answer	Marks	Guidance	
2	(v)		Require $\frac{\partial g}{\partial x}=\frac{\partial g}{\partial y}=\frac{\partial g}{\partial z}$ $(=\lambda)$	M1	*Allow M1 if $\lambda=1$*	
			$2x+2z=4y+2z=-2z+2x+2y+4$	A1	FT	
			$x=2y,\quad y=2z-2$			
			$(4z-4)^2+2(2z-2)^2-z^2+...+4z-3=0$	M1	Obtaining equation in one variable	
			$5z^2-8z+3=0$	A1	Or $5x^2+8x=0$ or $5y^2+4y=0$	Or $\lambda^2-4=0$
			Points $(0,\ 0,\ 1)$ and $(-1.6,\ -0.8,\ 0.6)$	M1	Obtaining at least one point	
			$k=0+0+1$ or $k=-1.6-0.8+0.6$	M1	Obtaining a value of k	*Implies previous M1 if values of x, y, z not seen*
			$k=1,\quad -1.8$	A1A1 **[8]**		
3	(i)		$\frac{dx}{d\theta}=3a\cos^2\theta\sin\theta,\quad \frac{dy}{d\theta}=3a\sin^2\theta\cos\theta$	B1		
			$\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2=(3a\sin\theta\cos\theta)^2(\cos^2\theta+\sin^2\theta)$	M1		
			$=(3a\sin\theta\cos\theta)^2$	A1		
			$s=\int 3a\sin\theta\cos\theta\,d\theta$	M1A1	FT	A1 requires workable integral form
			$=\frac{3}{2}a\sin^2\theta\quad(+c)$	A1		
			$s=0$ when $\theta=0\Rightarrow c=0$		Or $\int_0^\theta ...$ used	Required for final E1
			$\tan\psi=\frac{dy}{dx}=\frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta}$	M1		
			$=\tan\theta$	A1		
			Hence $\psi=\theta$ and $s=\frac{3}{2}a\sin^2\psi$	E1 **[9]**	Correctly shown	

Question			Answer	Marks	Guidance	
3	(ii)		$\rho = \dfrac{\mathrm{d}s}{\mathrm{d}\psi} = 3a\sin\psi\cos\psi$	M1A1		
			When $\theta = \dfrac{\pi}{6}$, $\psi = \dfrac{\pi}{6}$, $\rho = 3a\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$	M1		
			Radius of curvature is $\dfrac{3\sqrt{3}}{4}a$	A1		*Condone use of $\psi = \dfrac{\pi}{6}$ even if $\psi = \theta$ not established in (i)*
		OR	When $\theta = \dfrac{\pi}{6}$, $\dot{x} = \dfrac{9a}{8}$, $\dot{y} = \dfrac{3\sqrt{3}a}{8}$		M1 Obtaining second derivatives	
			$\ddot{x} = \dfrac{3\sqrt{3}a}{8}$, $\ddot{y} = \dfrac{15a}{8}$		A1	*May be implied by later work*
			$\rho = \dfrac{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y} - \ddot{x}\dot{y}} = \dfrac{\left(\frac{81a^2}{64} + \frac{27a^2}{64}\right)^{\frac{3}{2}}}{\left(\frac{9a}{8}\right)\left(\frac{15a}{8}\right) - \left(\frac{3\sqrt{3}a}{8}\right)\left(\frac{3\sqrt{3}a}{8}\right)}$		M1 Applying formula for ρ or κ	
			Radius of curvature is $\dfrac{3\sqrt{3}}{4}a$		A1	
			Normal vector is $\hat{\mathbf{n}} = \begin{pmatrix} -\sin\psi \\ \cos\psi \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$	M1	Obtaining gradient or normal vector	
				A1	Correct normal vector	Not necessarily unit vector
			$\mathbf{c} = \begin{pmatrix} a\left(1 - \frac{3\sqrt{3}}{8}\right) \\ \frac{1}{8}a \end{pmatrix} + \dfrac{3\sqrt{3}}{4}a\begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$	M1	Must use unit normal here	
			Centre of curvature is $\left(a\left(1 - \dfrac{3\sqrt{3}}{4}\right),\ \dfrac{5a}{4} \right)$	A1A1		
				[9]		

Question			Answer	Marks	Guidance	
3	(iii)		Surface area is $\int 2\pi x \,\mathrm{d}s$	M1		
			$= \int_0^{\frac{\pi}{3}} 2\pi a(1-\cos^3\theta)(3a\sin\theta\cos\theta)\,\mathrm{d}\theta$	A1	FT	*Correct limits required*
			$= 6\pi a^2 \int_0^{\frac{\pi}{3}} (\sin\theta\cos\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$			
			$= 6\pi a^2 \left[\frac{1}{2}\sin^2\theta + \frac{1}{5}\cos^5\theta \right]_0^{\frac{\pi}{3}}$	M1 A1A1	Integrating For $\frac{1}{2}\sin^2\theta$ and $\frac{1}{5}\cos^5\theta$	At least one trigonometric term Or equivalent
			$= \dfrac{87\pi a^2}{80}$	A1		
				[6]		
4	(i)		$\begin{array}{c\|cccc} P & 1 & 5 & 7 & 11 \\ \hline 1 & 1 & 5 & 7 & 11 \\ 5 & 5 & 1 & 11 & 7 \\ 7 & 7 & 11 & 1 & 5 \\ 11 & 11 & 7 & 5 & 1 \end{array}$	B1		
			Table shows closure	B1		
			Identity is 1	B1		
			All elements are self-inverse	B1	*Condone no mention of inverse of 1*	
				[4]		
4	(ii)		$(xy)(y^{-1}x^{-1})$	M1	Or $(y^{-1}x^{-1})(xy)$	
			$= x(yy^{-1})x^{-1} = xex^{-1} = xx^{-1} = e$	E1		
			So $y^{-1}x^{-1}$ is the inverse of xy			
				[2]		
4	(iii)		$a^{-1} = a,\ b^{-1} = b,\ c^{-1} = c,\ c^{-1} = (ab)^{-1} = b^{-1}a^{-1}$	M1	For any one of these	
			Hence $c = ba$	E1		
				[2]		

Question			Answer	Marks	Guidance	
4	(iv)		$bc = b(ba)$ $bc = ea = a$ $ac = a(ab) = eb = b$ $cb = a,\quad ca = b$	M1 E1 E1 B1 **[4]**	Or $ba = c \Rightarrow a = b^{-1}c$	Any correct first step
4	(v)		$\begin{array}{c\|cccc} R & e & a & b & c \\ \hline e & e & a & b & c \\ a & a & e & c & b \\ b & b & c & e & a \\ c & c & b & a & e \end{array}$ *R* is closed Hence *R* is a subgroup Same pattern as *P*; hence *R* and *P* are isomorphic	B1 M1 E1 E1 **[4]**	No need to mention identity or inverses *Dependent on B1 (only)*	
4	(vi)		$\begin{array}{c\|cccccccc} \text{Element} & A & B & C & D & E & F & G & H \\ \hline \text{Order} & 2 & 2 & 2 & 2 & 1 & 4 & 2 & 4 \end{array}$	B3 **[3]**	Give B1 for 3 correct; B2 for 6 correct	
4	(vii)		$\{E, A\}, \{E, B\}, \{E, C\}, \{E, D\}, \{E, G\}$ $\{E, F, G, H\}$ $\{E, A, B, G\}$ $\{E, C, D, G\}$	B2 B1 B1 B1 **[5]**	Ignore $\{E\}$ and *T* in the marking Give B1 for 3 correct	Deduct 1 mark (from this B2) for each subgroup of order 2 given in excess of five Deduct 1 mark (from this B1B1B1) for each subgroup of order 3 or more given in excess of three

Question			Answer	Marks	Guidance	
5			***Pre-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	
	(i)		*A* and *E* are reflecting barriers	B1 **[1]**		
5	(ii)		$\mathbf{P}=\begin{pmatrix}0&0.4&0&0&0\\1&0&0.4&0&0\\0&0.6&0&0.4&0\\0&0&0.6&0&1\\0&0&0&0.6&0\end{pmatrix}$	B2 **[2]**	Give B1 for three columns correct	
5	(iii)		$\mathbf{P}^{10}=\begin{pmatrix}0.1378&.&.&.&.\\0&.&.&.&.\\0.4689&.&.&.&.\\0&.&.&.&.\\0.3933&.&.&.&.\end{pmatrix}$	M1 M1	For $\mathbf{P}^{10}$ or $\mathbf{P}^{9}$ Using first or second column of $\mathbf{P}^{10}$ or $\mathbf{P}^{9}$	
			Possible positions are $A,\ C,\ E$ $\mathrm{P}(A)=0.1378,\ \mathrm{P}(C)=0.4689,\ \mathrm{P}(E)=0.3933$	B1 A1 **[4]**		
5	(iv)		$\mathbf{P}^{13}=\begin{pmatrix}0&\ldots\\0.3162&\ldots\\0&\ldots\\0.6838&\ldots\\0&\ldots\end{pmatrix},\ \mathbf{P}^{2}=\begin{pmatrix}.&.&.&.&.\\.&0.64&.&.&.\\.&.&.&.&.\\.&.&.&0.84&.\\.&.&.&.&.\end{pmatrix}$	M1	Using first or second column of $\mathbf{P}^{13}$ or $\mathbf{P}^{12}$, and diagonal elements from $\mathbf{P}^{2}$	
			$0.3162\times0.64+0.6838\times0.84$	M1	Allow M1 for $0.1301\times0.4+0.4651\times0.48+0.4048\times0.6$	$(=0.5182)$
			Probability is 0.7768	A1 **[3]**		

Question			Answer	Marks	Guidance	
5	(v)		$\mathbf{P}^{15}=\begin{pmatrix} . & .... \\ . & .... \\ . & .... \\ 0.6882 & .... \\ . & .... \end{pmatrix}$, $\mathbf{P}^{17}=\begin{pmatrix} . & .... \\ . & .... \\ . & .... \\ 0.6904 & .... \\ . & .... \end{pmatrix}$ After 17 steps	M1 M1 A1 **[3]**	Considering powers of **P** Appropriate element exceeding 0.69	
5	(vi)		Limit of $\mathbf{P}^{2n}$ is $\begin{pmatrix} 0.1231 & 0 & 0.1231 & 0 & 0.1231 \\ 0 & 0.3077 & 0 & 0.3077 & 0 \\ 0.4615 & 0 & 0.4615 & 0 & 0.4615 \\ 0 & 0.6923 & 0 & 0.6923 & 0 \\ 0.4154 & 0 & 0.4154 & 0 & 0.4154 \end{pmatrix}$ Limit of $\mathbf{P}^{2n+1}$ is $\begin{pmatrix} 0 & 0.1231 & 0 & 0.1231 & 0 \\ 0.3077 & 0 & 0.3077 & 0 & 0.3077 \\ 0 & 0.4615 & 0 & 0.4615 & 0 \\ 0.6923 & 0 & 0.6923 & 0 & 0.6923 \\ 0 & 0.4154 & 0 & 0.4154 & 0 \end{pmatrix}$	B2 B2 **[4]**	Give B1 for any non-zero element correct to 3 dp Give B1 for any non-zero element correct to 3 dp	*SC* If $\mathbf{P}^{2n}$ and $\mathbf{P}^{2n+1}$ interchanged, award B1B1
5	(vii)		Expected time at A is 50×0.1231 A: 6.2 B: 15.4 C: 23.1 D: 34.6 E: 20.8 (min)	M1 A2 **[3]**	Condone 100×0.1231 for M1 Give A1 for one correct	
5	(viii)		Expected number is $\frac{1}{0.4}-1$ $=1.5$ Expected time is $1+1.5\times 2$ $=4$ minutes	M1 A1 M1 A1 **[4]**	For $\frac{1}{0.4}$ For 1.5×2	

Question			Answer	Marks	Guidance	
5			***Post-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	
	(i)		A and E are reflecting barriers	B1 **[1]**		
5	(ii)		$\mathbf{P}=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 & 0 \\ 0 & 0.4 & 0 & 0.6 & 0 \\ 0 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	B2 **[2]**	Give B1 for three rows correct	
5	(iii)		$\mathbf{P}^{10}=\begin{pmatrix} 0.1378 & 0 & 0.4689 & 0 & 0.3933 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$	M1 M1	For $\mathbf{P}^{10}$ or $\mathbf{P}^{9}$ Using first or second row of $\mathbf{P}^{10}$ or $\mathbf{P}^{9}$	
			Possible positions are $A,\ C,\ E$ $\mathrm{P}(A)=0.1378,\ \mathrm{P}(C)=0.4689,\ \mathrm{P}(E)=0.3933$	B1 A1 **[4]**		

Question			Answer	Marks	Guidance	
5	(iv)		$\mathbf{P}^{13} = \begin{pmatrix} 0 & 0.3162 & 0 & 0.6838 & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$ $\mathbf{P}^{2} = \begin{pmatrix} . & . & . & . & . \\ . & 0.64 & . & . & . \\ . & . & . & . & . \\ . & . & . & 0.84 & . \\ . & . & . & . & . \end{pmatrix}$	M1	Using first or second row of $\mathbf{P}^{13}$ or $\mathbf{P}^{12}$, and diagonal elements from $\mathbf{P}^{2}$	
			$0.3162 \times 0.64 + 0.6838 \times 0.84$	M1	Allow M1 for $0.1301 \times 0.4 + 0.4651 \times 0.48 + 0.4048 \times 0.6$	$(= 0.5182)$
			Probability is 0.7768	A1 **[3]**		
5	(v)		$\mathbf{P}^{15} = \begin{pmatrix} . & . & . & 0.6882 & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}, \mathbf{P}^{17} = \begin{pmatrix} . & . & . & 0.6904 & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$	M1 M1	Considering powers of **P** Appropriate element exceeding 0.69	
			After 17 steps	A1 **[3]**		

Question			Answer	Marks	Guidance	
5	(vi)		Limit of $\mathbf{P}^{2n}$ is $\begin{pmatrix} 0.1231 & 0 & 0.4615 & 0 & 0.4154 \\ 0 & 0.3077 & 0 & 0.6923 & 0 \\ 0.1231 & 0 & 0.4615 & 0 & 0.4154 \\ 0 & 0.3077 & 0 & 0.6923 & 0 \\ 0.1231 & 0 & 0.4615 & 0 & 0.4154 \end{pmatrix}$	B2	Give B1 for any non-zero element correct to 3 dp	
			Limit of $\mathbf{P}^{2n+1}$ is $\begin{pmatrix} 0 & 0.3077 & 0 & 0.6923 & 0 \\ 0.1231 & 0 & 0.4615 & 0 & 0.4154 \\ 0 & 0.3077 & 0 & 0.6923 & 0 \\ 0.1231 & 0 & 0.4615 & 0 & 0.4154 \\ 0 & 0.3077 & 0 & 0.6923 & 0 \end{pmatrix}$	B2 **[4]**	Give B1 for any non-zero element correct to 3 dp	*SC* If $\mathbf{P}^{2n}$ and $\mathbf{P}^{2n+1}$ interchanged, award B1B1
5	(vii)		Expected time at A is 50×0.1231 A: 6.2 B: 15.4 C: 23.1 D: 34.6 E: 20.8 (min)	M1 A2 **[3]**	Condone 100×0.1231 for M1 Give A1 for one correct	
5	(viii)		Expected number is $\frac{1}{0.4} - 1$ $= 1.5$ Expected time is $1 + 1.5 \times 2$ $= 4$ minutes	M1 A1 M1 A1 **[4]**	For $\frac{1}{0.4}$ For 1.5×2	

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4757 Further Applications of Advanced Mathematics (FP3)

General Comments

Each question on this paper contained parts which were accessible to most of the candidates, and other parts which presented a significant challenge. This resulted in a wide range of marks providing good discrimination between the candidates. The most popular options were Q1 (vectors) and Q2 (multi-variable calculus) which were both attempted by the majority of candidates, and the least popular was Q3 (differential geometry) which was attempted by about a quarter of the candidates.

Comments on Individual Questions

1) In part (i) the vector product was usually evaluated correctly. A fairly common error was to divide by a common factor (usually 10) to make the numbers more manageable, obtaining an answer which was a multiple of the correct one. Almost all candidates knew how to use the vector product to find the equation of the hillside.

   In part (ii) most candidates applied a valid method for finding the shortest distance from a point to a line, usually the standard formula based on the magnitude of a vector product. There was some confusion with the formulae for the distance from a point to a plane and for the distance between two lines.

   In part (iii) the method for finding the shortest distance between two skew lines was very well understood, and most candidates carried it out accurately.

   In part (iv) the most efficient method was to set up three equations from the coordinates of the two intersecting lines, from which the point of intersection and the value of p could both be found. Some candidates used a separate method for finding p based on the shortest distance between the lines being zero. About half the candidates obtained both the point and the value of p correctly, with careless errors, often leading to very awkward numerical values, spoiling many answers.

2) In parts (i) and (ii) the partial derivatives and the equation of the normal line were usually given correctly. A few candidates gave the equation of the tangent plane instead of the normal line.

   In part (iii) candidates needed to relate both the change h in g and the length of PQ to the parameter in the equation of the normal line. Most candidates considered just one of these (usually the first), with about 10% of candidates earning full marks in this part.

   In part (iv) almost all candidates realised that $\partial g/\partial x = \partial g/\partial y = 0$. To proceed beyond this it was necessary to use the equation of the surface, and very many candidates did not do this; a common error was to assume that $\partial g/\partial z = 1$.

   In part (v) most candidates knew that the three partial derivatives were equal. It was then necessary to substitute into the equation of the surface to find the coordinates of the appropriate points on the surface and hence the values of k. Many candidates made algebraic and numerical slips in this process, and many did not use the equation of the surface at all, usually taking the common value of the partial derivatives to be 1.

3) In part (i) the relevant techniques were well understood, and most candidates made substantial progress. About a half of the attempts completed the derivation of the intrinsic equation successfully. A minor error made by many candidates was not considering the constant of integration when obtaining the expression for s.

In part (ii) the radius of curvature was usually given correctly; those who differentiated the intrinsic equation were much more likely to obtain the right answer than those who used the formula involving second derivatives. The method for finding the centre of curvature was well understood, and about one third of the attempts obtained both coordinates correctly. Many candidates made minor slips in this process, particularly when finding a unit normal vector.

In part (iii) many candidates produced a correct integral expression for the surface area, and about a quarter of the attempts completed the integration to obtain the correct value.

4) In part (i) most candidates produced a satisfactory proof that P is a group.
The group algebra in parts (ii), (iii) and (iv) was quite well done; the main error was to state that the group is commutative without justifying this assertion.

In part (v) almost all candidates completed the composition table correctly. Very many candidates did not state that R is closed, which is necessary to show that R is a subgroup of Q. Most candidates gave a satisfactory reason why R is isomorphic to P.
Almost all candidates gave the orders of the elements in part (vi) correctly; the most common error was to give the order of the identity E as 2 instead of 1.

In part (vii) most candidates listed the five subgroups of order 2 and the cyclic subgroup of order 4. Part (v) of this question shows how subgroups containing four self-inverse elements can be identified; very many candidates did not include the two subgroups of this type.

5) Almost all candidates identified the reflecting barriers in part (i) and wrote down the transition matrix in part (ii) correctly.

In part (iii) most candidates understood how to find the probabilities. A significant number gave what were in fact the positions and probabilities after 9 steps instead of 10.
About half of the candidates knew how to answer part (iv). Parts (v) and (vi) were generally well answered.

Part (vii) was also quite well understood. Some candidates multiplied the equilibrium probabilities by 100 instead of 50, and many confused this with the problem of finding run lengths.

In part (viii) many candidates found the expected number of movements correctly. Finding the expected time was more challenging.

GCE Mathematics (MEI)

			Max Mark	90% cp	a	b	c	d	e	u
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	66	60	53	47	41	34	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	90	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	73	65	57	50	43	36	0
		UMS	100	90	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	66	61	53	46	39	32	0
		UMS	100	90	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	61	54	47	40	34	28	0
		UMS	100	90	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	68	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	90	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	67	63	56	50	44	38	0
		UMS	100	90	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	63	56	49	42	35	29	0
		UMS	100	90	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	66	61	55	49	43	38	0
		UMS	100	90	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	63	56	49	42	35	28	0
		UMS	100	90	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	62	56	50	44	39	34	0
		UMS	100	90	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	52	46	40	34	29	24	0
		UMS	100	90	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	63	55	47	39	32	25	0
		UMS	100	90	80	70	60	50	40	0